Eigenvalues and Eigenvectors

nerd (i guess)

August 14, 2021

1 Jesse what the fuck are you talking about.

For a given linear transformation (a mapping between 2 different vector spaces that preserves the operations of addition and scalar multiplication), there may be some vectors that stay on the same span before and after the transformation (i.e it doesn't change direction)

A vectors that behaves this way is called an Eigenvector, and how much they are scaled up or down is a scalar called an Eigenvalue

Of course, the zero vector is guaranteed to have these properties, but the term here SPECIFICALLY refers to non-trivial, non-zero vectors.

2 How are these determined?

Linear transformations are represented by matrices, so we can assume some linear transformation represented by matrix A, with our eigenvector $\vec{v}$ and our eigenvalue λ. Given our definition of eigenvalues and eigenvectors, we can say:

$$A\vec{v} = \lambda I\vec{v}$$

Which can be simplified to:

$$(A - \lambda I)\vec{v}$$

$= 0$

Now. knowing that we exclude the possibility of a zero vector being the eigenvector, we come to the conclusion that:

$$det(A - \lambda I) = 0$$

This is the final equation we need! using this, we can solve for the eigenvalues, and then use that to find the eigenvectors!

3 Examples

Assume we want to find the eigenvalues and eigenvectors of a linear transformation represented by the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ We can use $det(A - \lambda I) = 0$ to say that:

$$det(\begin{bmatrix} 1-\lambda & 0 \\ 2 & 3-\lambda \end{bmatrix})$$
$$= (1-\lambda)(3 - -\lambda) - (0)(2)$$
$$= (1-\lambda)(3-\lambda) = 0$$

Now, we could solve for the eigenvectors but I'm tired lmao fuck you (jk ily)